Crocheting

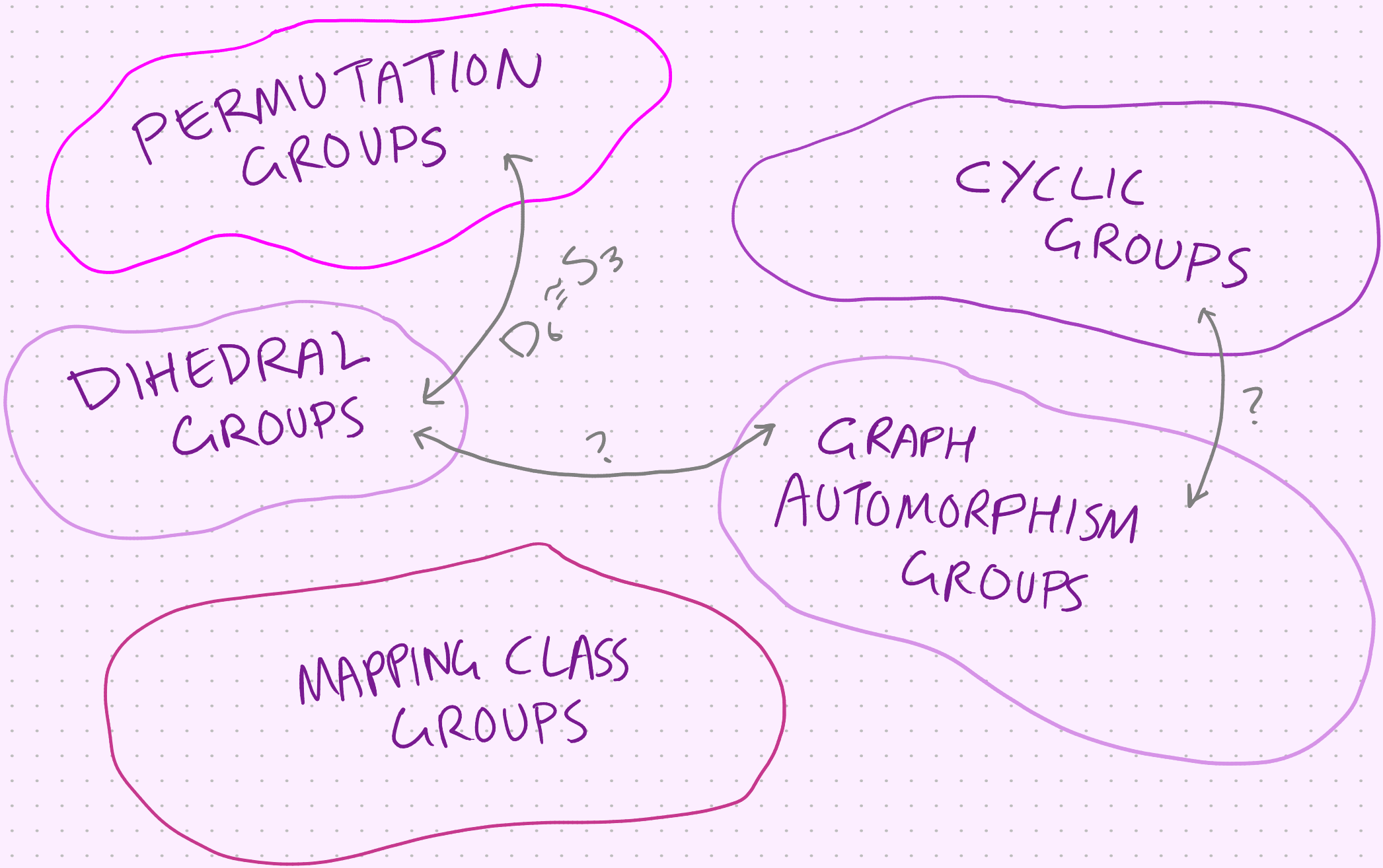
AN ISOMORPHISM

Between the SYMMETRY GROUPS of the KLEIN QUARTIC

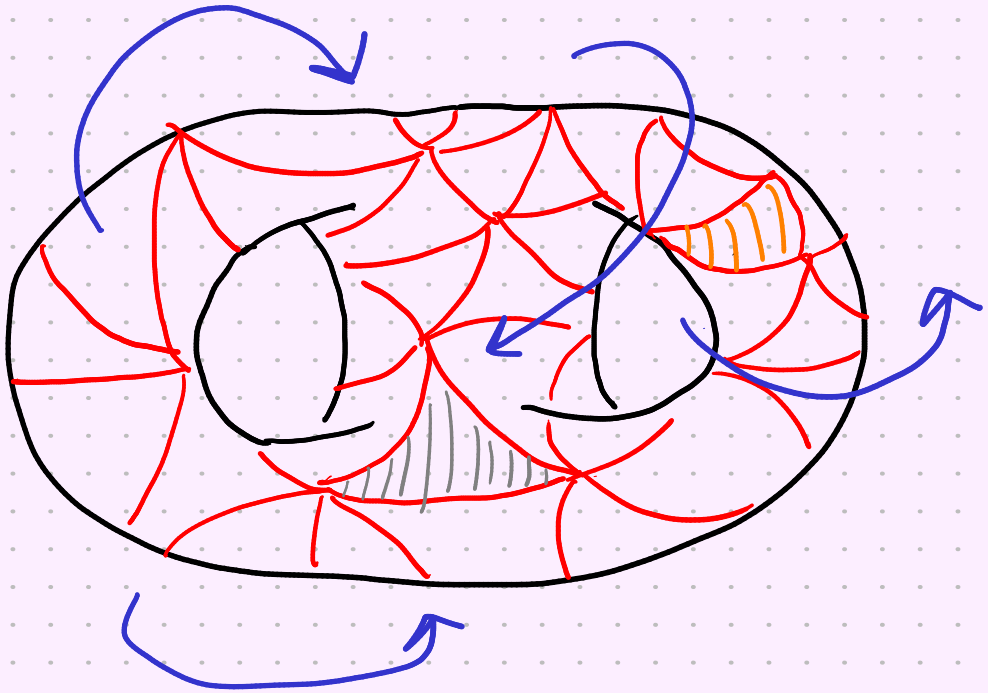
AND

FANO PLANE

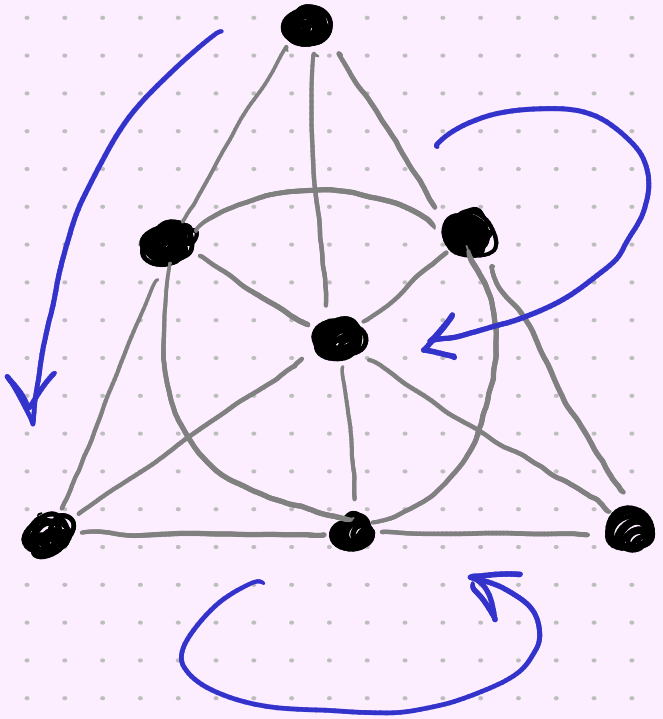
EXCEPTIONAL ISOMORPHISMS



SYMMETRY GROUP OF KLEIN QUARTIC

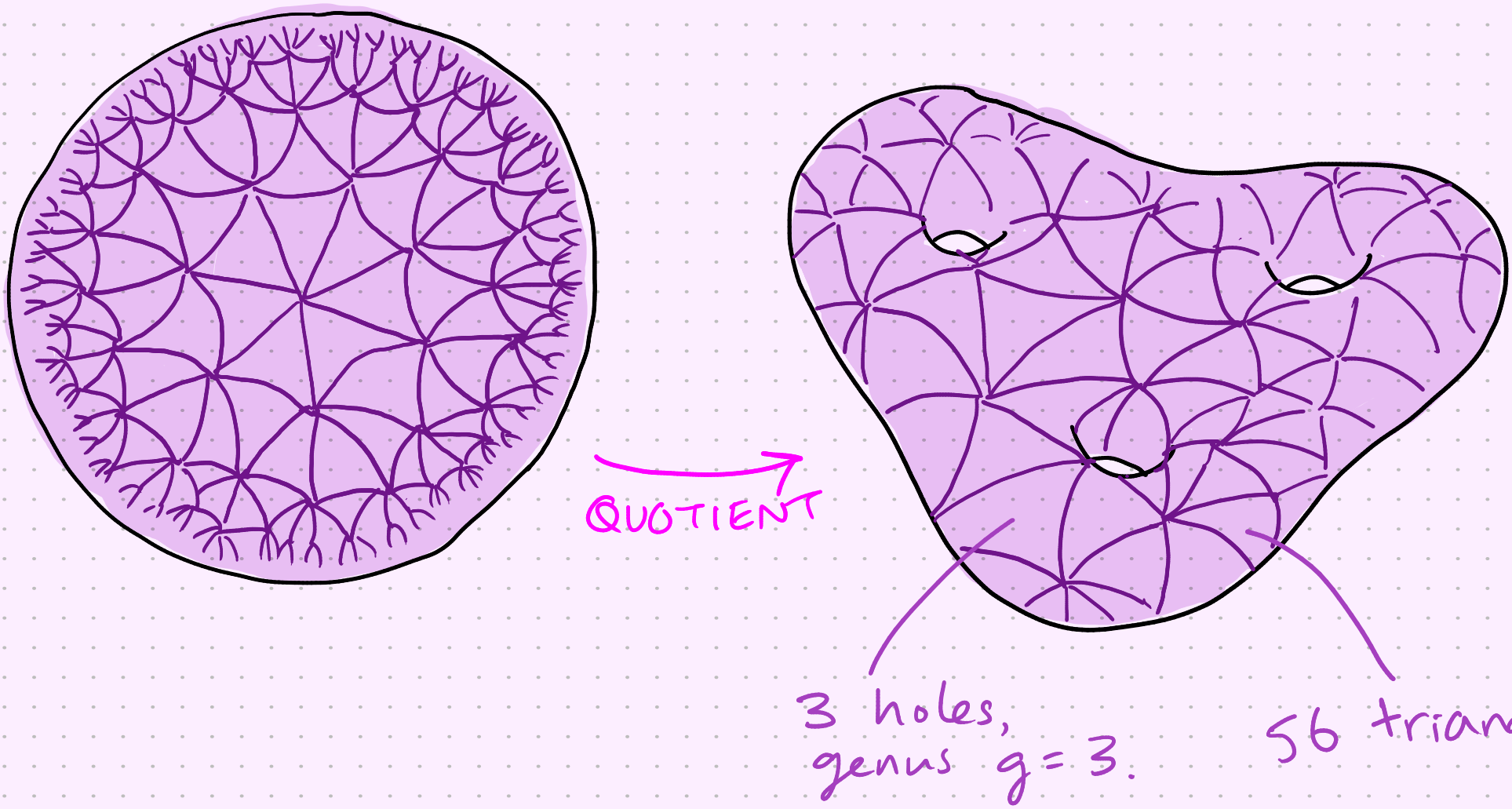


SYMMETRY GROUP OF FANO PLANE



THEY'RE THE SAME!

WHAT IS THE KLEIN QUARTIC?



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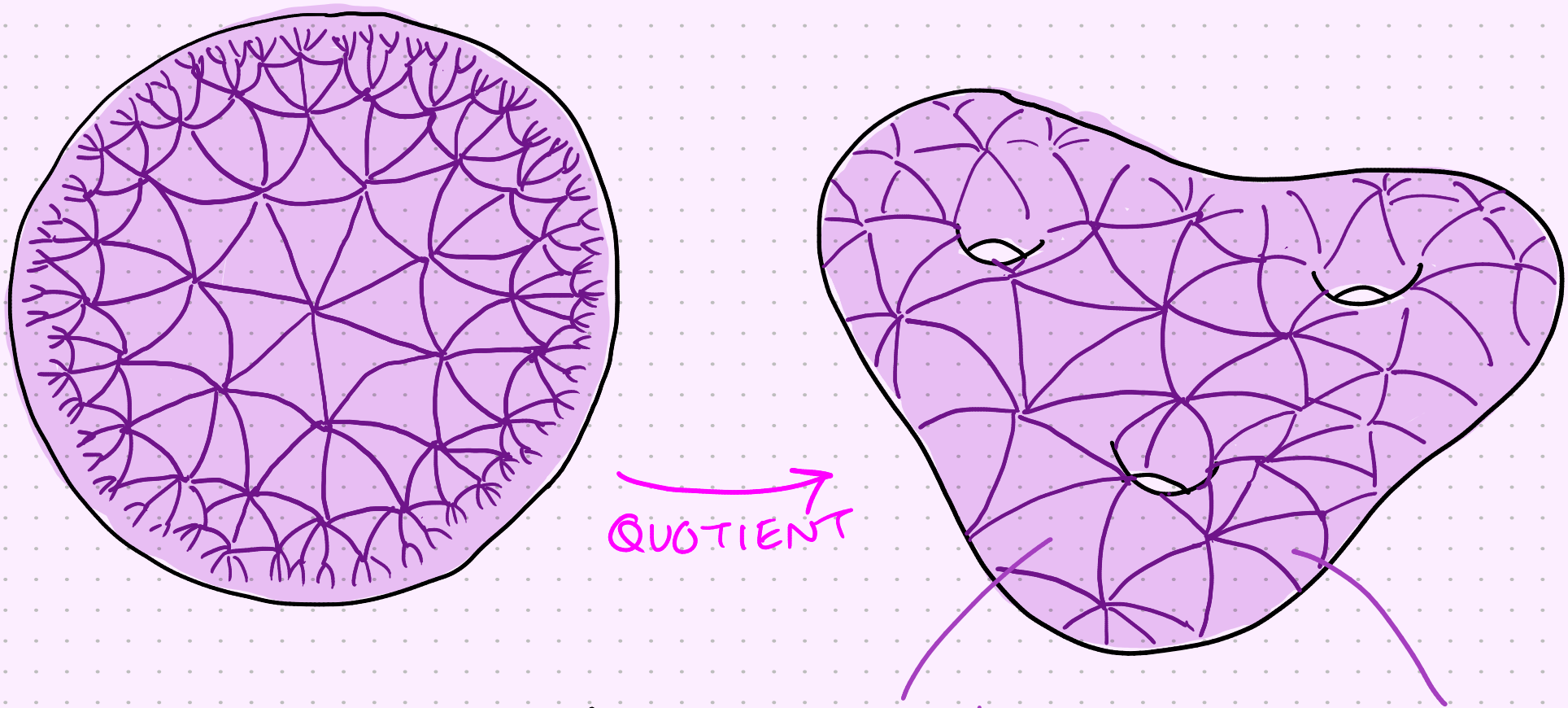
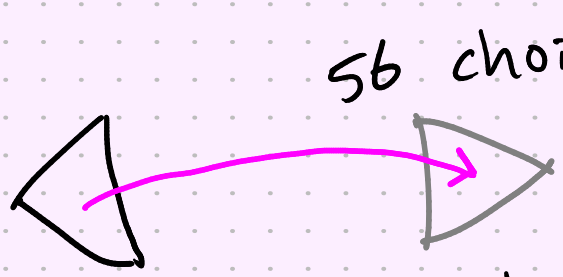
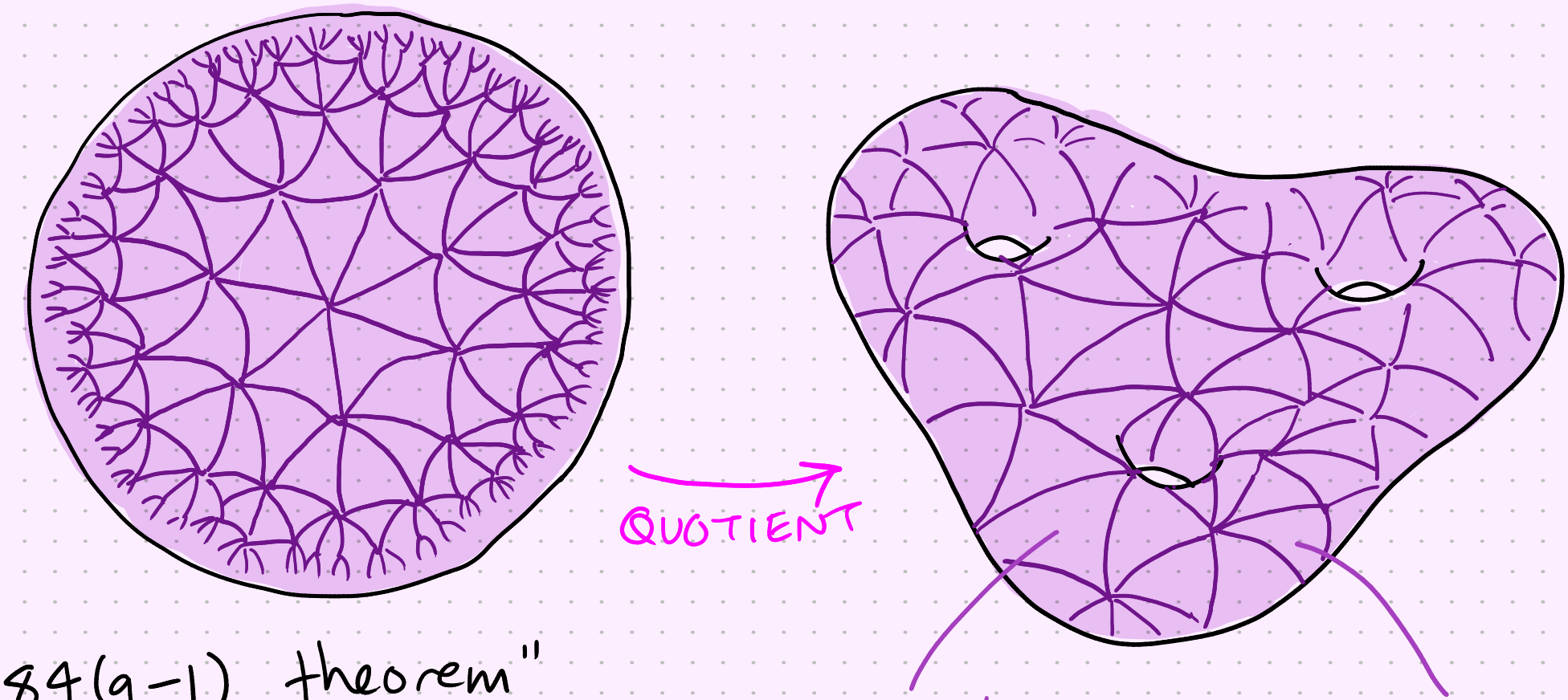




3 choices of rotation

$56 \times 3 = 168$ symmetries.

WHAT IS THE KLEIN QUARTIC?

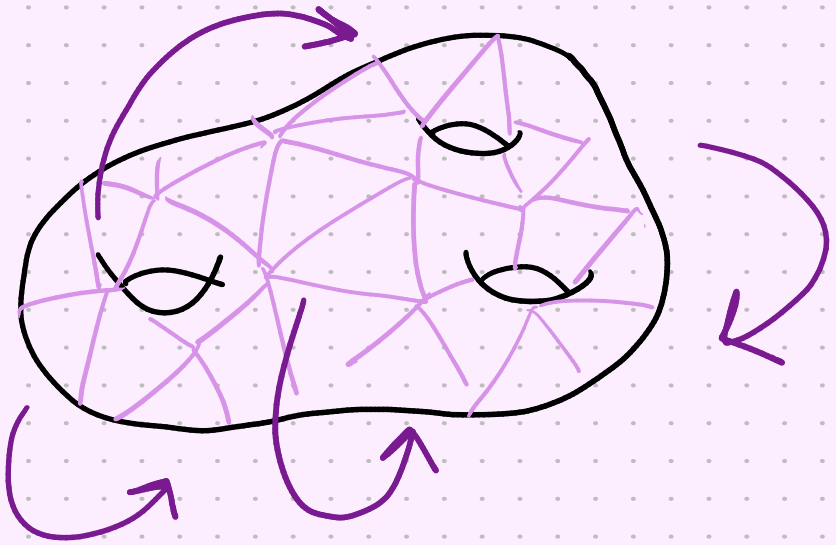


"84(g-1) theorem"

A hyperbolic surface of genus g has at most 84(g-1) symmetries.

The klein quartic has 168 = 84(3-1) symmetries!

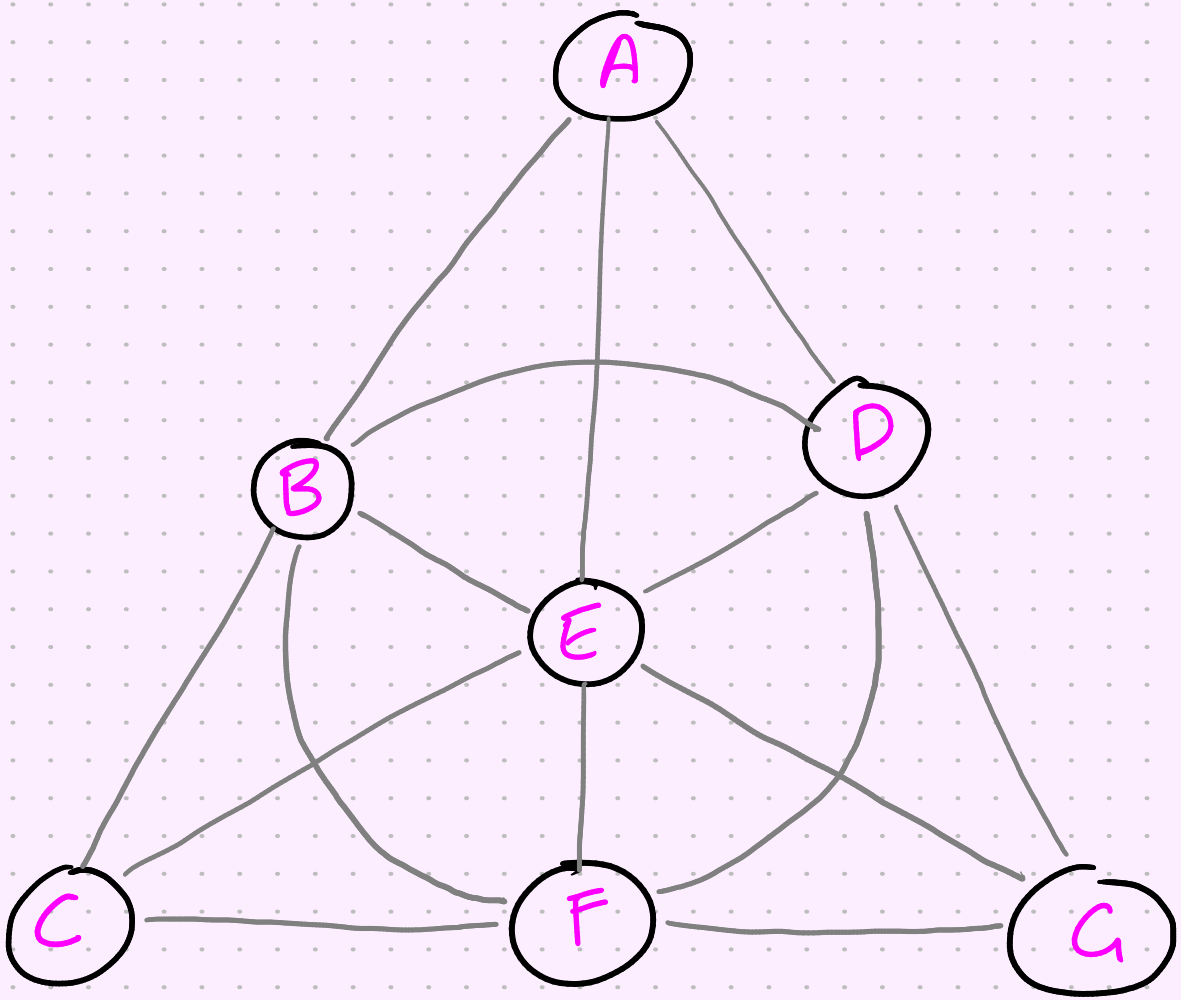
SYMMETRIES OF THE KLEIN QUARTIC



$= PSL(2,7)$

Certain 2×2 matrices

WHAT IS THE FANO PLANE?



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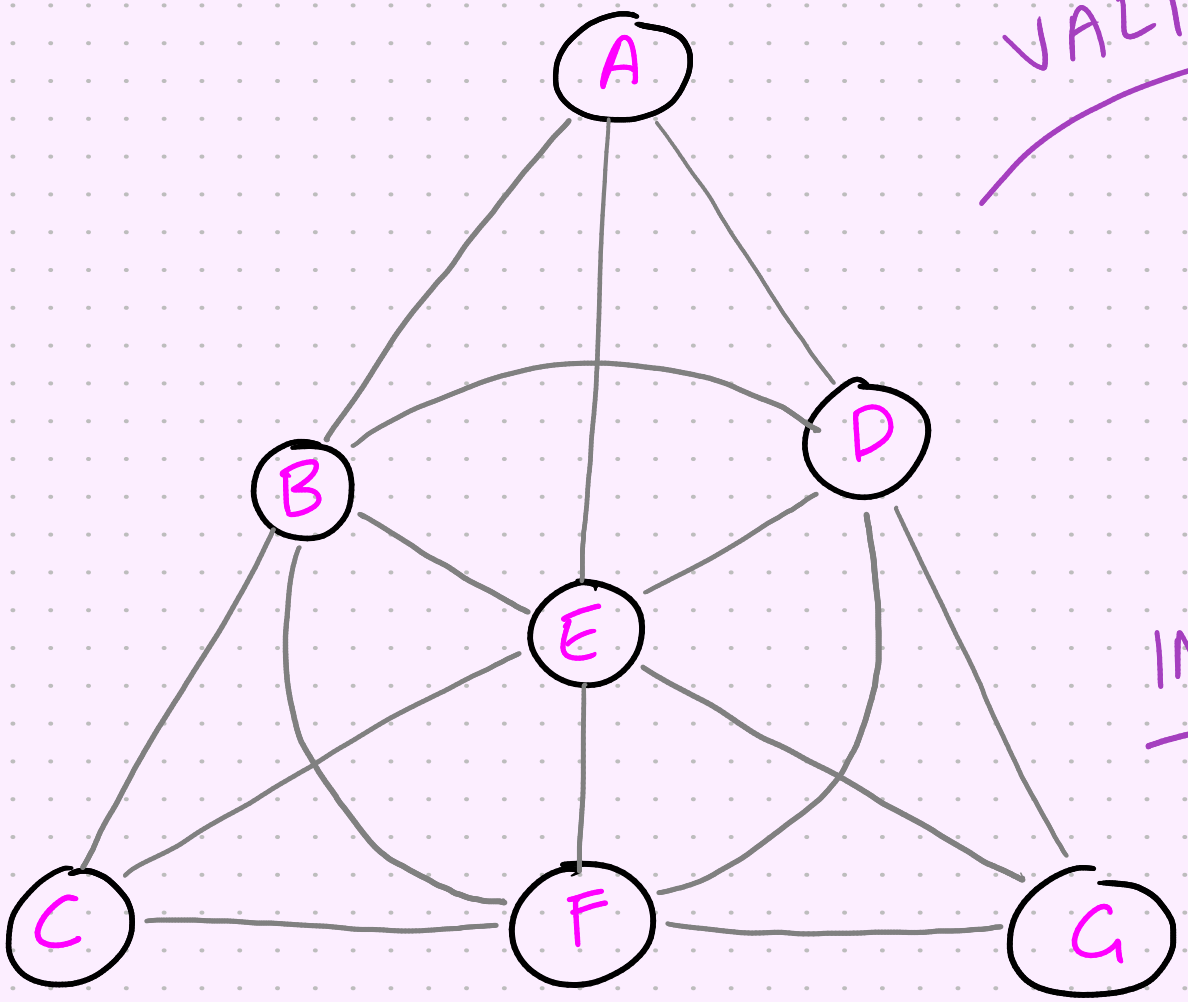
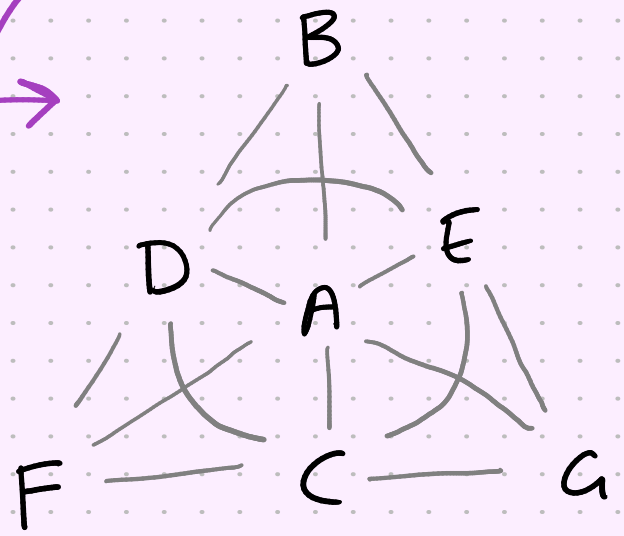
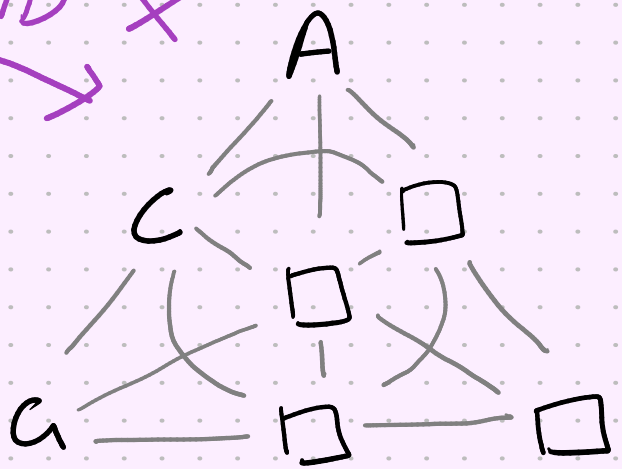


SYMMETRIES

VALID ✓

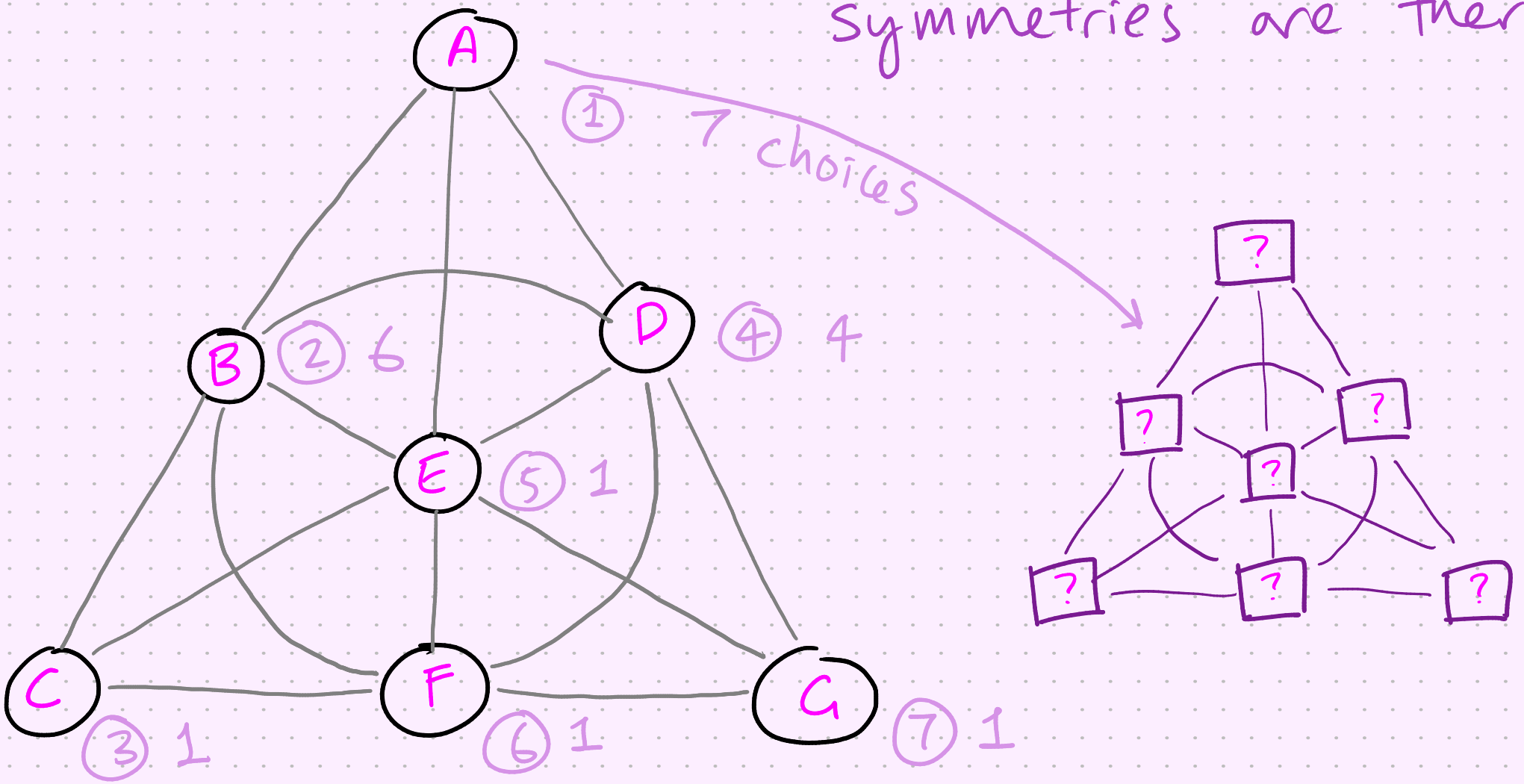


INVALID ✗



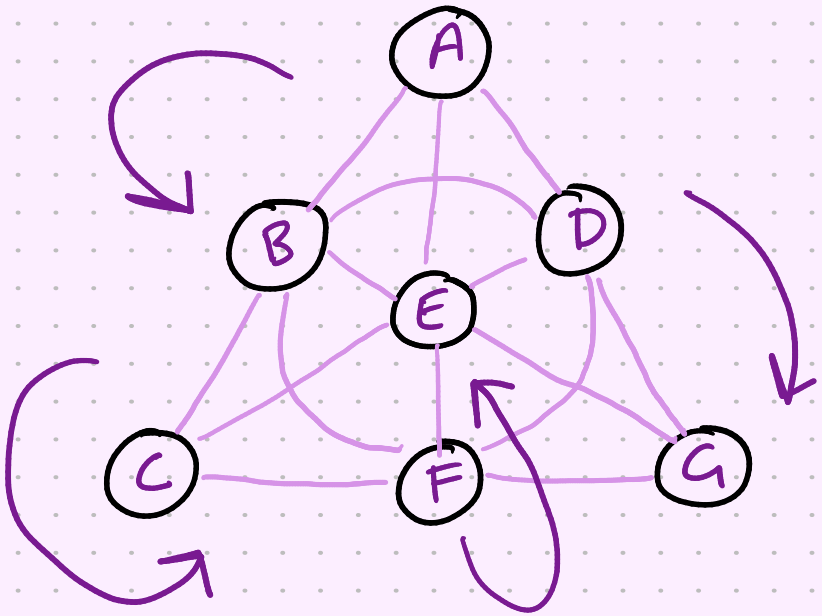
WHAT IS THE FANO PLANE?

How many symmetries are there?



$7 \times 6 \times 4 = 168$ symmetries again!

SYMMETRIES OF THE FANO PLANE

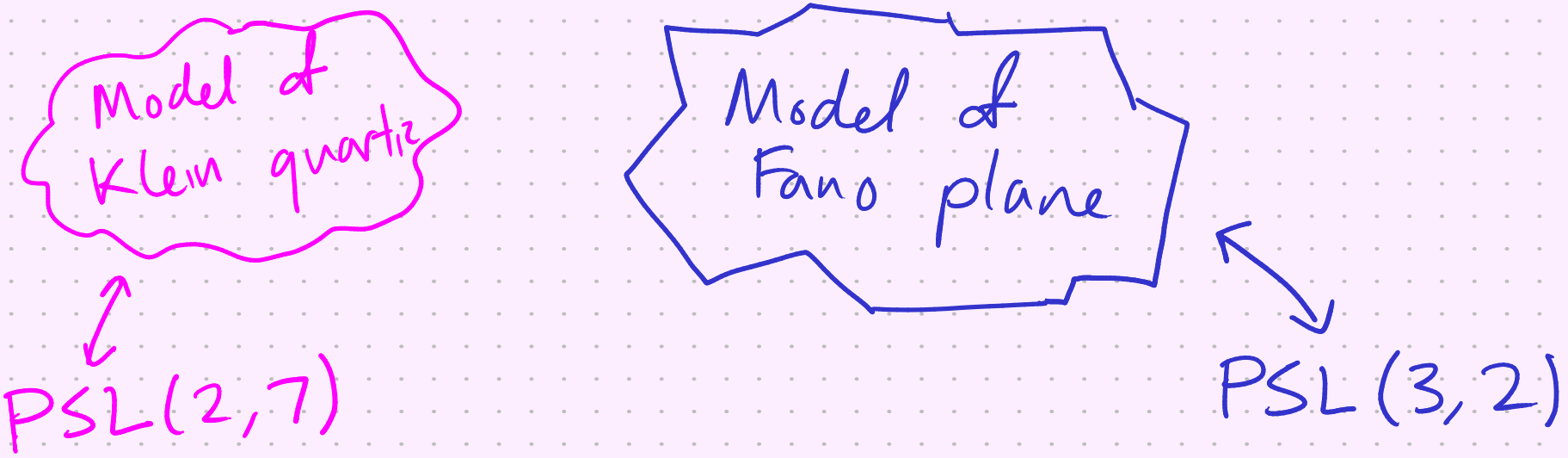


$= PSL(3,2)$

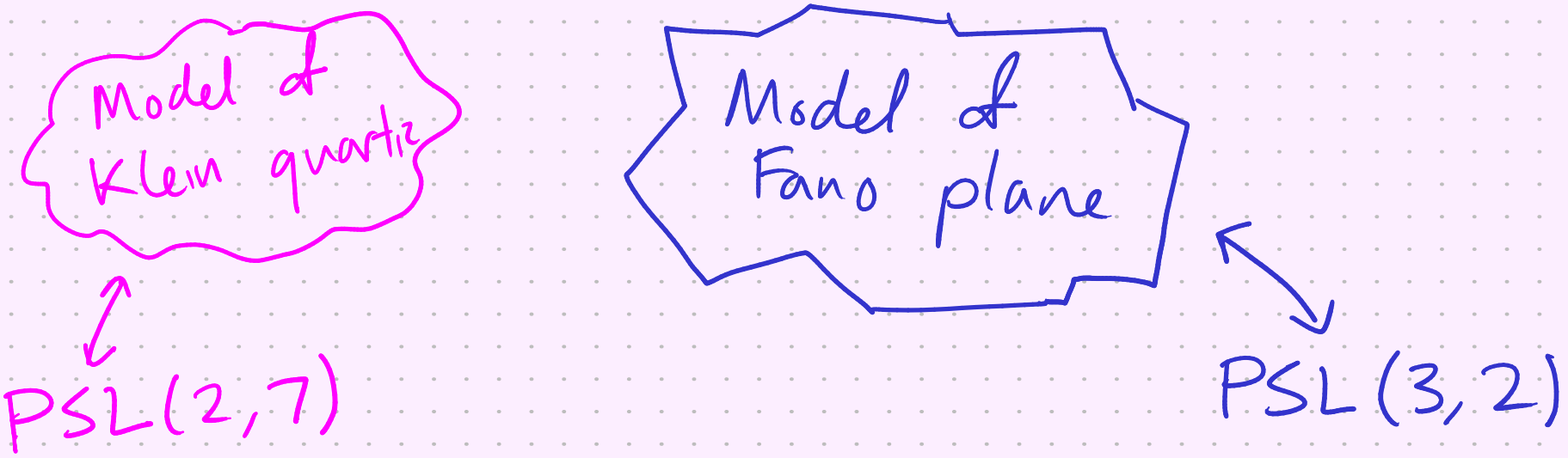
Certain 3×3 matrices

EXCEPTIONAL ISOMORPHISM

$PSL(2,7) \cong PSL(3,2)$. WHY?



EXCEPTIONAL ISOMORPHISM
$PSL(2,7) \cong PSL(3,2)$. WHY?



What if we made a single model of both the Klein quartic and Fano plane?

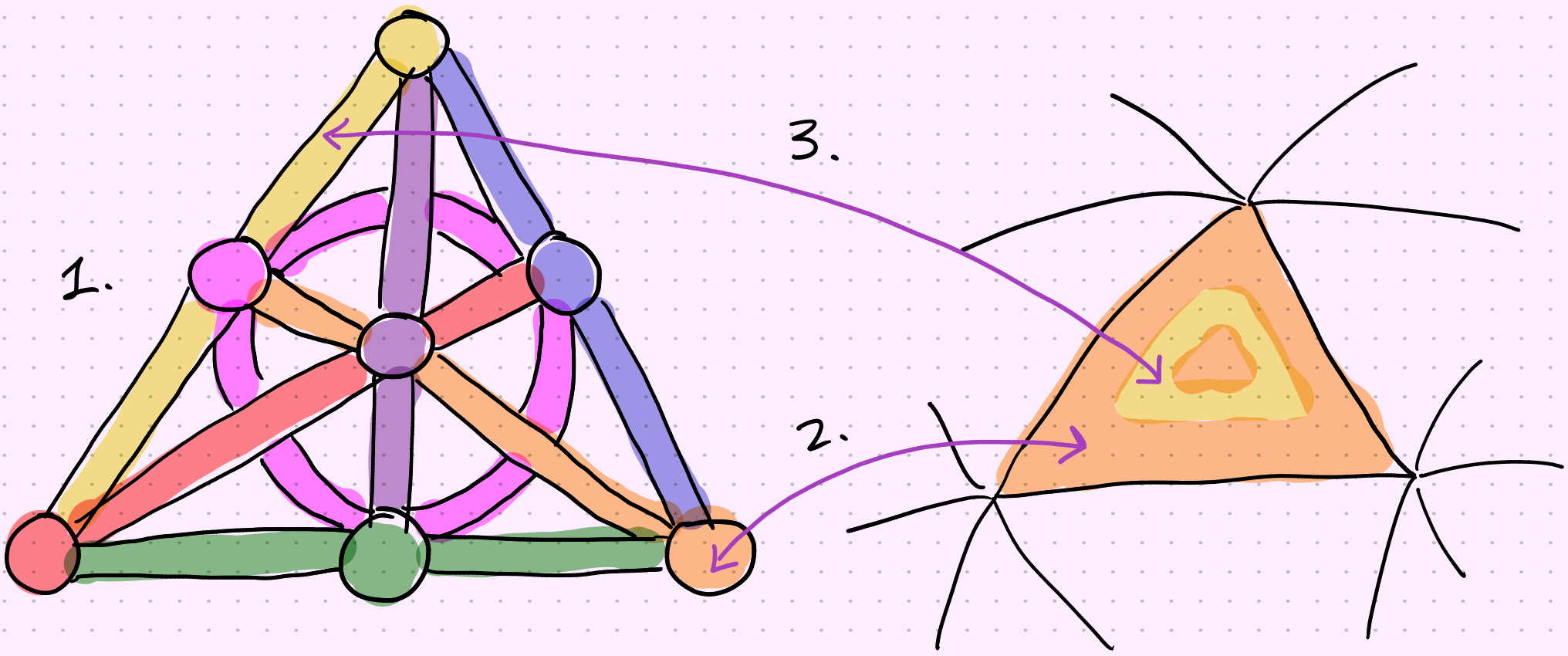
WHAT DOES THE MODEL LOOK LIKE?

The surface and tiling are literally the Klein quartic!

The colours encode the Fano plane!

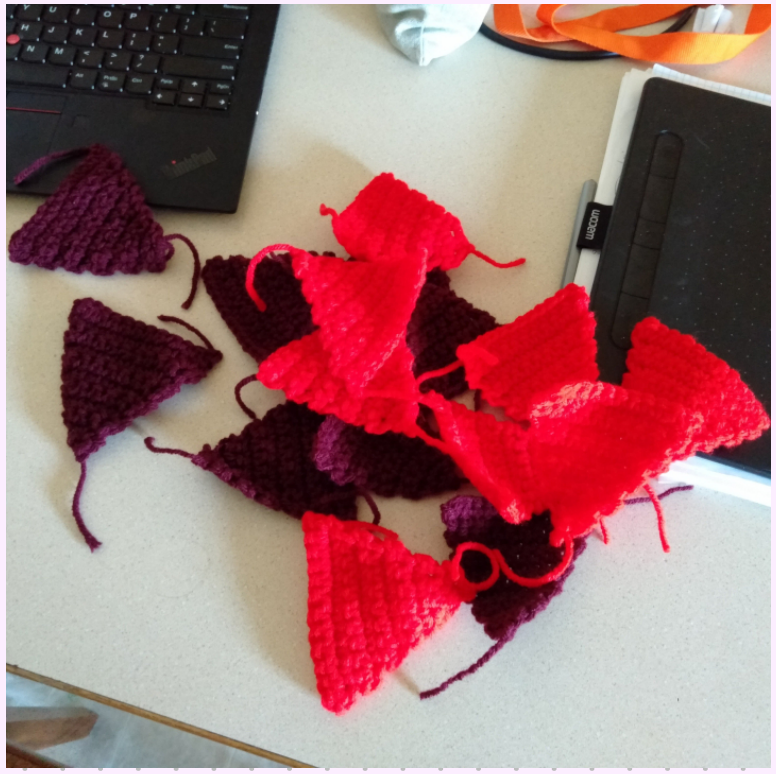
SYMMETRIES OF THE KLEIN QUARTIC ARE PRECISELY SYMMETRIES OF THE COLOURS (FANO PLANE)

HOW DID I ENCODE THE FANO PLANE?



1. Colour the Fano plane
2. Colour the faces of the klein quartic to represent vertices
3. Add an extra colour to the faces to represent edges.

HOW DID I MAKE THE MODEL?



1. Make 56 triangles! 8 of each colour.

I used single crochet. The stitches are the most uniform, giving me equilateral triangles.

2. Pull it all together!
"Quilt" it with black yarn.

I underestimated how dense it would become at the end. Should've used thinner yarn.